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How children perceive fractals: Hierarchical self-similarity and cognitive development

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ABSTRACT

The ability to understand and generate hierarchical structures is a crucial component of human cognition, available in language, music, mathematics and problem solving. Recursion is a particularly useful mechanism for generating complex hierarchies by means of self-embedding rules. In the visual domain, fractals are recursive structures in which simple transformation rules generate hierarchies of infinite depth. Research on how children acquire these rules can provide valuable insight into the cognitive requirements and learning constraints of recursion.

Here, we used fractals to investigate the acquisition of recursion in the visual domain, and probed for correlations with grammar comprehension and general intelligence. We compared second ($n = 26$) and fourth graders ($n = 26$) in their ability to represent two types of rules for generating hierarchical structures: Recursive rules, on the one hand, which generate new hierarchical levels; and iterative rules, on the other hand, which merely insert items within hierarchies without generating new levels. We found that the majority of fourth graders, but not second graders, were able to represent both recursive and iterative rules. This difference was partially accounted by second graders' impairment in detecting hierarchical mistakes, and correlated with between-grade differences in grammar comprehension tasks. Empirically, recursion and iteration also differed in at least one crucial aspect: While the ability to learn recursive rules seemed to depend on the previous acquisition of simple iterative representations, the opposite was not true, i.e., children were able to acquire iterative rules before they acquired recursive representations. These results suggest that the acquisition of recursion in vision follows learning constraints similar to the acquisition of recursion in language, and that both domains share cognitive resources involved in hierarchical processing.

1. Introduction

The ability to represent and generate complex hierarchical structures is one of the hallmarks of human cognition. In many domains, including language, music, problem-solving, action-sequencing, and spatial navigation, humans organize basic elements into higher-order groupings and structures (Badre, 2008; Chomsky, 1957; Hauser, Chomsky, & Fitch, 2002; Nardini, Jones, Bedford, & Braddick, 2008; Unterrainer & Owen, 2006; Wohlschlager, Gattis, & Bekkering, 2003). This ability to encode the relationship between items (words, people,

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etc.) and the broader structures where these items are embedded (sentences, corporations, etc.), affords flexibility to human behavior. For example, in action sequencing, humans are able to change, add, or adapt certain basic movements to particular contexts, while keeping the overall structure (and goals) of canonical motor procedures intact (Wohlschlager et al., 2003).

The ability to process hierarchical structures develops in an interesting way. Young children seem to have a strong bias to focus on the local information contained within hierarchies. For instance, in the visual-spatial domain, while attending to a big square composed of small circles, children have a tendency to identify the small circles faster and easier than they can identify the big square (Harrison & Stiles, 2009; Poirel, Mellet, Houdé, & Pineau, 2008). This local-oriented strategy to process hierarchical stimuli is similar to non-human primates (Fagot & Tomonaga, 1999; Spinozzi, De Lillo, & Truppa, 2003), and it usually precludes adequate hierarchical processing. Conversely, in human adults a global bias develops, in which global aspects of hierarchical structures are processed first, and where the contents of global information interfere with the processing of local information (Bouvet, Rousset, Valdois, & Donnadieu, 2011; Hopkins & Washburn, 2002). This ability to represent items-in-context is one of the pre-requisites of hierarchical processing. In other domains such as in language, children display equivalent impairments: they seem to grasp the meaning of individual words, and of simple adjacent relationships between them, but display difficulties in extracting the correct meaning of sentences containing more complex constructions (Dąbrowska, Rowland, & Theakston, 2009; Friederici, 2009; Roeper, 2011). This progressive development in the ability to integrate local and global information within hierarchies seems to be associated with brain maturational factors (Friederici, 2009; Moses et al., 2002), but also with the amount of exposure to the particular kinds of structures that children are asked to process (Roeper, 2011).

In this study, we are interested in investigating a particular aspect of hierarchical processing, which is the ability to encode hierarchical self-similarity. Hierarchies can be generated and represented using processes that establish relationships of dominance and subordination between different items (Martins, 2012). Some of these processes are depicted in Fig. 1. For instance, 'iterative rules' (Fig. 1A) can be used to represent the successive addition of items to a structure, such as the addition of beads to a string to form a necklace. 'Embedding rules' can also be used to generate hierarchies by embedding one or more items into a structure so that they depend on another item (Fig. 1B). For example, in an army hierarchy, two brigades can be incorporated into a division. Finally, we can also use 'recursive embedding rules' to generate and represent hierarchies. Recursive embedding, or simply 'recursion', is the process by which we embed one or more items as dependents of another item of the same category (Fig. 1C). For example, in a compound noun we can embed a noun inside another noun, as in [[student] committee]. As we can see from Fig. 1, recursion is interesting and unique because it allows the generation of multiple hierarchical levels with a single rule.

One important notion to retain here is that recursion can be defined either as a "procedure that calls itself" or as the property of "constituents that contain constituents of the same kind" (Fitch, 2010; Pinker & Jackendoff, 2005). Frequently, we find an isomorphism between procedure and structure, i.e., recursive processes often generate recursive structures. However, this isomorphism does not always occur (Lobina, 2011; Luuk & Luuk, 2010; Martins, 2012). In this manuscript we explicitly focus on the level of representation, i.e., we focus on detecting *what* kind of information individuals can represent (i.e. hierarchical self-similarity), rather than on *how* this information is implemented algorithmically.

The ability to perceive similarities across hierarchical levels (i.e. hierarchical self-similarity) can be advantageous in parsing complex structures (Koike & Yoshihara, 1993). On the one hand, representing several levels with a single rule obviously reduces memory demands. On the other hand, this property allows the generation of new (previously absent) hierarchical levels without the need to learn or develop new rules or representations. This ability to represent hierarchical self-similarity, and to use this information to make inferences allows all the cognitive advantages postulated as being specifically afforded by 'recursion' (Fitch, 2010; Hofstadter, 1980; Martins, 2012; Penrose, 1989), namely the possibility to achieve infinity from finite means (Hauser et al., 2002).

One famous class of recursive structures is the fractals. Fractals are structures that display self-similarity (Mandelbrot, 1977), so that they appear geometrically similar when viewed at different scales. Fractals are produced by simple rules that, when applied iteratively to their own output, can generate complex hierarchical structures. Since the same kind of representation can be used at different levels of depth, simple rules suffice to represent the entirety of the structure. An example of a process generating a visuo-spatial fractal is depicted in Fig. 2. Here, a simple recursive rule adds a triad of smaller hexagons around each bigger hexagon. Since the relations between successive hierarchical levels are kept constant, individuals who are able to generate mental representations of recursion can make inferences about new (previously absent) hierarchical levels (Martins, 2012). This is the principle that we use in our investigation (For more details, see Appendix A). Our goal was to investigate how the ability to represent hierarchical self-similarity develops in the visual domain, and how this ability can be predicted by individual differences in intelligence, grammar comprehension and general visual processing.

The ability to represent hierarchical self-similarity has been empirically tested in the syntactic domain and in the visual domain (Martins & Fitch, 2012; Roeper, 2007). However, the developmental aspects of this ability have only been investigated in language (Roeper, 2011). In the next sections we briefly review what is currently known, and why it is important to extend this analysis to the visual-spatial domain.
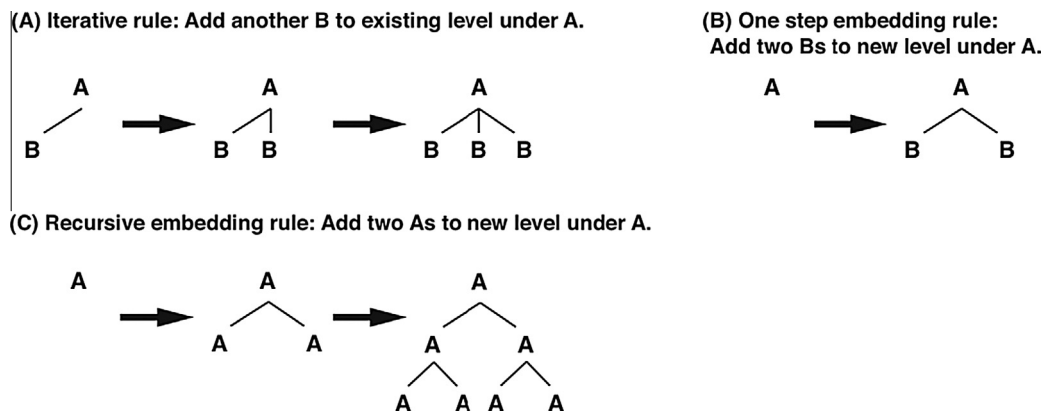
(A) Iterative rule: Add another B to existing level under A.

(B) One step embedding rule: Add two Bs to new level under A.

(C) Recursive embedding rule: Add two As to new level under A.

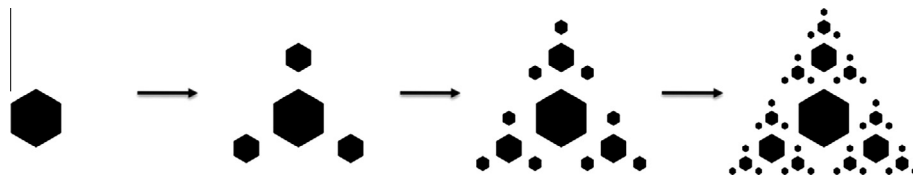**Fig. 1.** Examples of rules used to generate hierarchical relationships.

Fig. 2. Recursive process generating a visual fractal.

1.1. Hierarchical self-similarity and language

Within the domain of language, recursion seems to be universally used (Reboul, 2012), and although rare in common speech (Laury & Ono, 2010), most language users are likely to have generated recursive sentences (for instance, compound nouns such as "[[[student] film]] committee]"). The widespread use of recursion in syntax has lead to the influential hypothesis that recursion would be part of a computational module specific to language (Hauser et al., 2002). In its strongest version, the thesis 'minimalist program' postulates recursion as the central operation of most syntactic processes (Chomsky, 2010). Within this theory, the usage of recursion in other domains would be dependent on the activation of linguistic resources. It is thus essential to empirically investigate the ability to acquire recursion in non-linguistic domains and examine its relation to linguistic capacity.

The development of recursion remains controversial. In English, children as young as 7-years-old are able to generate novel recursive structures, despite being exposed to a very limited recursive input (Berwick, Pietroski, Yankama, & Chomsky, 2011; Roeper, 2009). They can also discriminate well-formed recursive constructions at the age of 3 (Alegre & Gordon, 1996). This has been taken as evidence that children are able to represent recursion *a priori*. Studies concerning the processing of child directed speech suggest that the presence of recursive rules as Bayesian priors better explain the acquisition of language than priors without recursion (Perfors, Tenenbaum, Gibson, & Regier, 2010). Bayesian priors can be understood as analogous to *a priori* expectations that bias individuals to interpret stimuli in a certain way. In the case of language, Bayesian priors can bias individuals to interpret sentences as having a particular syntactic structure and not others. In this Bayesian framework, although the ability to represent recursion is assumed to be present in the cognitive repertoire of young children, its explicit use in particular kinds of constructions may require experience with enough examples from those specific kinds. This experience may rapidly lead to the development of abstract representations, if a process of overgeneralization occurs (Perfors, Tenenbaum, Griffiths, & Xu, 2011a; Perfors, Tenenbaum, & Regier, 2011b). Consistent with this framework, the ability to represent recursion becomes available at different ontogenetic stages for different syntactic categories (Alegre & Gordon, 1996; Roeper, 2007; Roeper, 2011). Initially, children tend to interpret linguistic hierarchies as non-recursive (Roeper, 2011), before they substitute these representations with more abstract (recursive) ones (Dickinson, 1987). This substitution process occurs if non-recursive representations become insufficient.

In sum, there are two main factors which can influence the ontogenetic development of the ability to represent hierarchical self-similarity. The first factor is a general process of brain maturation, which could impose hard limits on the kinds of information children are able to encode. Adult-like brain connectivity does not occur until the age of 8–9 (Friederici, 2009; Power, Fair, Schlaggar, & Petersen, 2010), and this brain connectivity pattern seems to enhance the ability to understand hierarchical structures (both recursive and non-recursive). The second factor concerns experience, and the cumulative acquisition of constructions of increased abstraction (from non-recursive to recursive). In the current study we were interested in investigating the contribution of these factors in the

acquisition of recursion in a non-linguistic domain. We developed a visuo-spatial paradigm using fractal stimuli to which children are not normally exposed. Thus, we could assess the ability to acquire novel recursive representations in a domain (visual fractals) to which children are less likely to have strong prior expectations than in the domain of language.

1.2. Current study

Here, we investigated whether the ability to represent structural self-similarity in visual hierarchies (fractals) followed a developmental time course similar to recursion in language, and occurred under similar learning constraints. We decided to compare two groups of children – second graders (7- to 8-year-olds) and fourth graders (9- to 10-year-olds) – which seem to differ in their ability to understand hierarchical and recursive structures in the linguistic domain (Friederici, 2009; Miller, Kessel, & Flavell, 1970). Differences between these groups have also been reported within the visual domain: children below the age of 9 seem to have a strong bias to focus on local visual information (Harrison & Stiles, 2009; Poirel et al., 2008), which as we have discussed, can affect normal hierarchical processing. Interestingly, also adults seem to display a strong local bias when exposed to novel and complex structures (Harrison & Stiles, 2009). This suggests that both maturational and experience factors play a role in determining visual processing strategies.

The paradigm that we used in this experiment was based on the one used by (Martins & Fitch, 2012): we present a series of images that build up a particular type of structure, incrementally, and the participants are asked to choose between two possible "completion" images that continue the pattern. In all cases, one of these two images is the "correct" continuation of the pattern in the first three images, and the other is a foil, quite similar but differing in some crucial respect. In the current experiment we did not provide response feedback, hence we could assess the natural cognitive abilities of the children, whether they were able to generalize the structural features of recursive stimuli. In this version of the task we also included stimuli with different levels of visual complexity, to evaluate the role of this factor, which is orthogonal to recursion itself, in the ability to extract hierarchical self-similarity principles in the visual domain. We included several categories of foils in order to prevent the use of simple heuristic strategies, and we added a second, non-recursive iterative task, with the same apparatus and experimental conditions as the ones described for the recursive task (Fig. 3).

Finally, we included a grammar comprehension and a non-verbal intelligence task in the test battery. With this setup we could investigate not only whether there are age differences in the ability to represent visual recursion and non-recursive iteration, but also the influence of several factors potentially related with these differences, namely: grammar comprehension, general intelligence and sensitivity to visual complexity. The inclusion of a grammar comprehension task in the procedure is also interesting to investigate whether there are domain-general factors involved in the processing of hierarchical

structures. If recursion is the core computational operation of syntactic operations (Chomsky, 2010), and if open-ended representations of self-similar hierarchies depend on the use of linguistic resources (Fitch, Hauser, & Chomsky, 2005; Hauser et al., 2002), we would expect to find a strong and specific correlation between grammar comprehension and visual recursion.[1] Alternatively: (1) if visual and linguistic hierarchical processing systems are completely independent, we would expect to find no correlation between these two domains; (2) if there are shared cognitive resources between language and visual hierarchical processing, not specifically related to recursion, we would expect to find a general correlation between grammar comprehension and both recursive and iterative visual tasks.
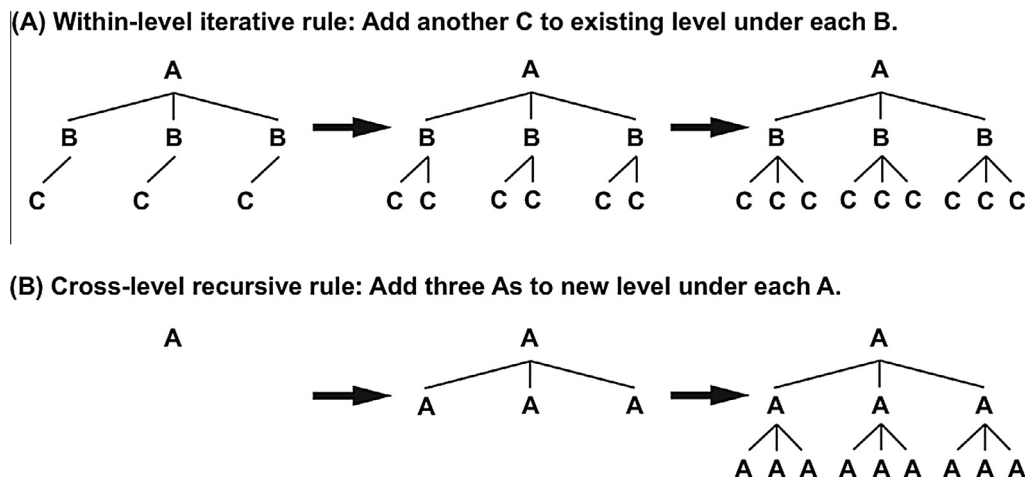
2. Methods

2.1. Participants

A total sample of 52 children took part in the study. They were all monolingual native speakers of German and were recruited from an elementary school in a middle-to-high socioeconomic neighborhood in Vienna (Austria). They were divided into two grade groups: 26 children (14 males) attended the second grade and were 7–8 years old ($M = 8;2$, range = 7;7–8;8); and 26 children (15 males) attended the fourth grade and were 9–10 years old ($M = 10;2$, range = 9;8–10;4). Exclusion criteria included bilingualism, known neurological and psychiatric medical history, developmental learning disorders, and visual or auditory impairment. Children's participation was conditional upon approval by their head teachers and teachers, and their own willingness to take part in the experiment. They were aware that they could withdraw from the experiment at any time without further consequences. Moreover, all parents provided written informed consent for their children's participation in the study, and all data was stored anonymously.

2.2. Procedure

Children were tested individually in a quiet room at their school, in a single session of approximately 45 min. During this session, participants performed 4 tasks: (1) The Visual Recursion Task (VRT), designed to assess the ability to represent recursive iterative processes in the visuo-spatial domain (Martins & Fitch, 2012); (2) The Embedded Iteration Task (EIT), designed to test the ability to represent non-recursive iterative processes in the visuo-spatial domain (Martins & Fitch, 2012); (3) The Test for Reception of Grammar (TROG-D), a grammatical comprehension task (Bishop, 2003; Fox, 2007); and (4) The Raven's

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[1] For instance, some authors have suggested that the usage of verbal resources to encode non-verbal information may increase the tractability of reasoning processes (Carruthers, 2002). This enhancement could be mediated by object semantics (Hitch et al., 1983), spatial referencing (Haun, Rapold, Janzen, & Levinson, 2011), item counting (Noël, 2009), sequential indexing of serial patterns (Duncan, 2010) or by analogical and associative reasoning (Fry & Hale, 2000). These processes could be necessary to represent visual fractals.

(A) Within-level iterative rule: Add another C to existing level under each B.

(B) Cross-level recursive rule: Add three As to new level under each A.

Fig. 3. Schematic depiction of the different representations required to perform our non-recursive iterative task (A), and our recursive task (B) (adapted from Martins et al. (2014)).

Coloured Progressive Matrices (CPM), a non-verbal intelligence task (Raven, Raven, & Court, 2010).

The whole testing procedure was divided into two parts, with a break of 5 min in between. The first part included VRT and EIT, as well as a specific training for these tasks, and the second part included TROG-D and CPM. The order of tasks in the first part was randomized and equally distributed: Within each grade group 13 children started the procedure with VRT and 13 children started the procedure with EIT. The order of tasks in the second part was fixed (TROG-D first and then CPM).

Both VRT and EIT were binary forced-choice paradigms, where children were asked to choose between two images. After the completion of the first two tasks, we asked 42 out of 52 children the following question: "How frequently were the two images in the bottom different? (a) Almost never, (b) Sometimes, or (c) Almost always?" We initiated this systematic questioning after the experiment had begun, due to the feedback that we got from some children, reporting perceiving no differences between the choice images. Unfortunately, it was not possible to retrieve the answer from the first 10 children.

2.3. Visual Recursion Task (VRT)

Test procedure. This task was adapted from the one used in (Martins & Fitch, 2012). In VRT, each trial began with the presentation of three images corresponding to the first three iterations (steps) of a fractal generation. These images were presented in the top half of the screen, sequentially from left to right ('Sequence images'; Fig. 4), with an interval of 2 s between the presentation of one image and the next. After the presentation of the first three iterations, two additional images were presented simultaneously in the bottom half of the screen ('Choice images'; Fig. 4). One image corresponded to the correct continuation of the recursive process that generated the first three fractals and the other corresponded to a foil (or 'incorrect' continuation). Participants were asked to touch the image

they considered as the correct continuation of the recursive process, and their response was captured using a touch-screen (Elo Touchsystems). The position of the 'correct' image (LEFT or RIGHT) was randomized. The same instructions were given (in German, and during training only) to all participants:

Instructions (English translation): "*Look, this picture puzzle works like this: Up at the top there are three pictures. And down below there are two pictures. You have to press on the correct picture down below. This is the first picture, this is the second picture, and this is the third picture. What is the correct **next** picture: this or that?* [Feedback: *Great, you got it right.* (or) *No, that was not correct. Look, this is the correct picture.*]"

After the initial instructions, each trial had a maximum duration of 30 s before a timeout. No visual or auditory feedback was given regarding whether the answer was correct or incorrect. The task comprised 27 trials, and had a total duration of about 12 min.

To test for effects of information processing constraints, we included stimuli with different degrees of visual complexity (complexity '3','4' and '5'). Furthermore, in order to control for the usage of simple visual heuristic strategies in VRT performance, we included several categories of foils ('Odd', 'Position' and 'Repetition'). For details on stimuli generation and stimuli categories, see Appendix A and Fig. 5. Overall, the combination of both 'visual complexity' and 'foils' categories resulted in 9 types of stimuli: Complexity 3, 4 and 5 with odd constituent foils; Complexity 3, 4 and 5 with positional error foils and Complexity 3, 4 and 5 with repetition foils. Exactly three examples of each type of stimuli were generated using the programming language Python, resulting in a total of 27 stimuli.

2.4. Embedded Iteration Task (EIT)

The second task was hierarchical but non-recursive, and was adapted from the one used in (Martins & Fitch, 2012). The principle underlying EIT is similar to VRT in the sense
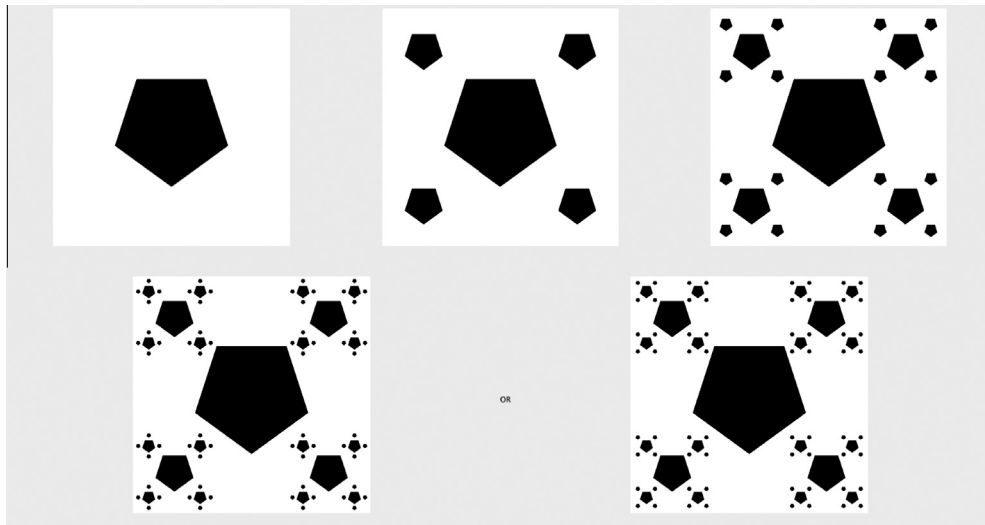
Fig. 4. Example of a typical Visual Recursion Task trial. Initially, the first three iterations of a fractal generation are depicted, sequentially, from left to right, with an interval of 2 s between iteration (top). On the presentation of each new image, the previous iterations remain visible on the screen, in the positions depicted in the figure. Then, while the first three iterations remain visible, two additional images are presented simultaneously in the bottom part of the screen, corresponding to the 'correct' fourth iteration (bottom right) and a foil (bottom left), and the participant chooses between them. In this example, the foil is a 'positional foil' (see Fig.5 for details on foils).
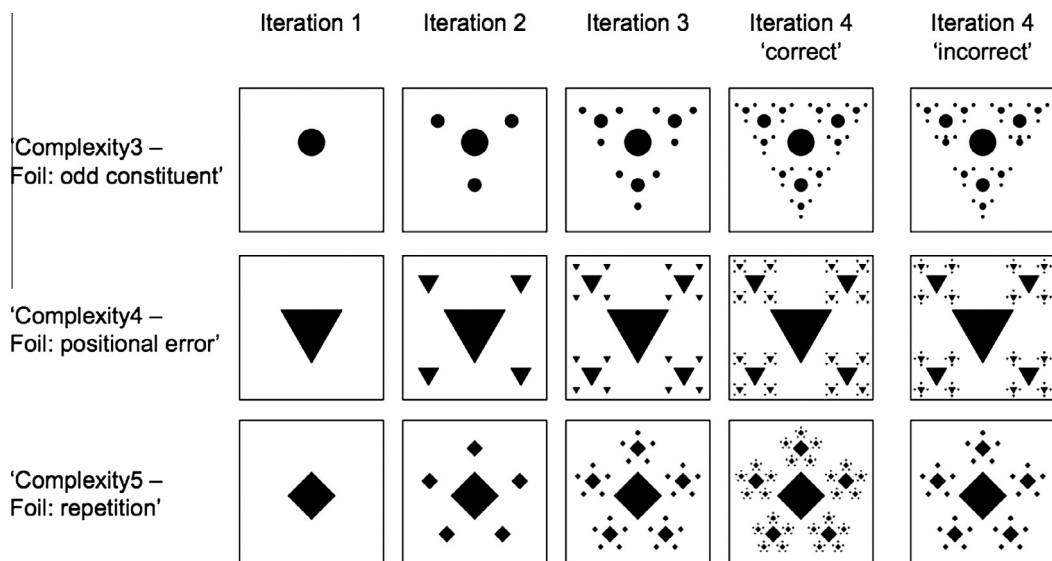
Fig. 5. Examples of fractals used in the Visual Recursion Task. The first four iterations of a fractal generation, as well as one foil ('incorrect' fourth iteration), were produced. There were different categories of 'visual complexity' – 3, 4 and 5 – and different categories of foils. In 'Odd constituent' foils two elements within the whole hierarchy were misplaced; in 'positional error' foils all elements within new hierarchical levels were internally consistent, but inconsistent with the previous iterations; in 'Repetition' foils no additional iterative step was performed after the third iteration.

that it involves an iterative procedure applied to hierarchical structures. However, EIT lacks recursive embedding. Instead, in EIT, additional elements are added to one pre-existing hierarchical structure, without producing new hierarchical levels (Fig. 6). As for VRT, an understanding of this iterative procedure is necessary to correctly predict the next iteration.

All the apparatus and experimental conditions for EIT were identical to the ones described for VRT, including number of trials, duration of each trial, 'visual complexity' categories, foil categories, and feedback conditions (see Appendix A).

2.5. Training

Prior to the beginning of the first part of the procedure (composed by VRT and EIT), a short training session was given. The goal of this training was to give children the
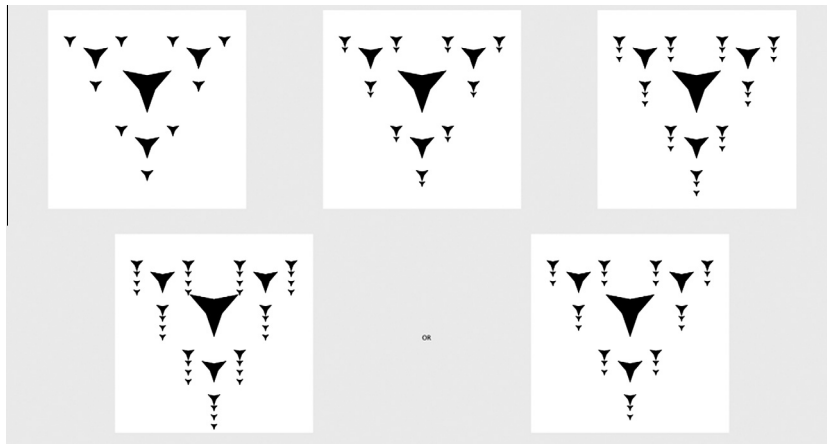
Fig. 6. Example of a typical embedded iteration task trial. In this example, the correct answer is on the bottom left. The foil on the bottom right is a "repetition foil" (see Fig.5 for details on foils).

opportunity to manipulate the touch-screen, and to introduce them to the specific environment of VRT and EIT trials before testing. Four training items were given: Two items followed an iterative rule, which was not hierarchical (see Appendix B for an example); one item was iterative and hierarchical, but not recursive (similar to the items of EIT); and the last item was iterative, hierarchical and recursive (similar to VRT). If participants provided an incorrect response, the same item was presented again until a correct response was provided. In case of repeated failure, the experimenter tried to motivate the child (during training only) by drawing his/her attention to the structure of the trial, and repeating the instructions if necessary.

2.6. TROG-D

TROG-D is a grammatical comprehension task designed for children aged 3 to 11 years. It is the German adaptation of the English *Test for Reception of Grammar* – TROG (Bishop, 2003) and was standardized using the data from 870 monolingual German-speaking children (Fox, 2007). The test consists of 84 test items grouped into 21 test blocks, with increasing difficulty: nouns, verbs, adjectives, 2-element sentences (SV), 3-element sentences (SVO), negation, prepositions ('in/on'), perfect tense, plural, prepositions ('above/below'), passive, personal pronouns (nominative), relative clauses (nominative), personal pronouns (accusative/dative), double object constructions, subordination ('while/after'), topicalization, disjunctive conjunctions ('neither-nor'), relative clauses (accusative/dative), coordination ('and'), subordination ('that'). Test items are presented in a four picture multiple-choice format with lexical and grammatical foils. The test procedure is as follows: The investigator reads aloud the test item to the child (e.g. relative clause (nominative): *Der Junge, **der** das Pferd jagt, ist dick* 'The boy, who is chasing the horse, is chubby'), and the task of the child is to point at the appropriate picture in the test booklet. Participants' responses are analyzed by test block ($N = 21$); in order for a test block

to be classified as correct, all responses within the test block have to be correct.

2.7. CPM

Raven's Coloured Progressive Matrices (CPM) is a nonverbal intelligence task (with a focus on logical reasoning) designed for children aged 5–11 years (Raven et al., 2010). The test consists of 36 test items grouped into 3 test sets (A, Ab, B), with 12 test items each. Test sets are arranged in a way so as to allow development of a consistent method of thinking; set A: completion of a single, continuous pattern, sets Ab and B: completion of discrete patterns. Test items are presented in a six-picture multiple-choice format. In each test item, the task of the child is to identify the missing element that completes a pattern and to point at it in the test booklet. Participants' responses are analyzed by test item ($N = 36$).

2.8. Predictions

Based on the previous discussion, our working hypothesis was that the ability to represent recursion becomes available at later ontogenetic stages than the ability to represent iteration, and that this difference is partially explained by biological development factors. Consequentially, our predictions were the following: (1) Fourth graders were expected to perform adequately in both recursive and iterative tasks, while second graders might be expected to do so in the non-recursive iterative task only; (2) Visual complexity was expected to play a role in performance, especially among the second graders; (3) The ability to perform adequately in the visual recursion task was expected to correlate in general with grammar comprehension abilities, and specifically with the comprehension of sentences with embedded clauses.

Alternatively, the potential to represent recursion might become available at the same ontogenetic stage as the potential to represent iteration. Differences in performance between recursive and iterative tasks might be

related not with effects of biological development, but with effects of cumulative exposure to visuo-spatial hierarchies (as it seems to occur in language). In other words, children may need to be exposed to a certain number of hierarchical examples generated iteratively before they are able to acquire recursive representations. If this were the case, we would expect to find strong task-order effects rather than between grade effects.

2.9. Analyses

Our overall goal was to assess children's ability to represent recursion and embedded iteration in the visual domain and to compare performance between second and fourth grade. Furthermore we investigated the effects of visual complexity, visual strategies (foil categories), task-order, grammar abilities and non-verbal intelligence.

In our data, we used the binomial variable VRT and EIT 'trial correctness' (correct/incorrect) as the dependent variable for regression models. When overall response data were not normally distributed (assessed using a Shapiro–Wilk test), we used non-parametric statistics. Simple response accuracy comparison between grades was performed with an unpaired Mann–Whitney U test. To assess whether each participant had VRT and EIT scores above chance, we first calculated the proportion of correct (and incorrect) answers that deviated significantly from chance using a Binomial test. Since we used a binary forced-choice task, the probability to score correctly due to chance was 50%. In a total of 27 test items, a number of correct answers equal or superior to 20 (i.e. a proportion of 0.74), or equal or inferior to 7 (i.e. a proportion of 0.26), is the number which differs significantly from chance (Binomial test, $p = 0.019$). The comparison between second and fourth grades, regarding children that scored above chance, was performed using a Chi-square test.

Finally, to assess the effects of visual strategies (foil categories), visual complexity, task-order, grammar abilities and non-verbal intelligence, we used a semiparametric regression technique called Generalized Estimating Equations (GEE), a technique useful when analyzing binomial data with within-subjects effects (Hanley, 2003). We created several models containing different variables: 'grade' and 'task-order' as between-subjects variables; 'task', 'foil category' and 'visual complexity' as within-subjects variables; and 'grammar' and 'intelligence' raw scores as covariates.

All analyses were performed with SPSS® 19.

3. Results

3.1. VRT

General overview: correct responses by grade. On average, the 26 children attending the fourth grade ($M = 0.80$, $SD = 0.21$) had a significantly higher proportion of correct responses in VRT than children attending the second grade ($M = 0.59$, $SD = 0.17$) (*Mann–Whitney U: z = −3.70, p < 0.001*; Fig. 7). Moreover, while 69.2% of fourth graders had a proportion of correct answers above chance, only
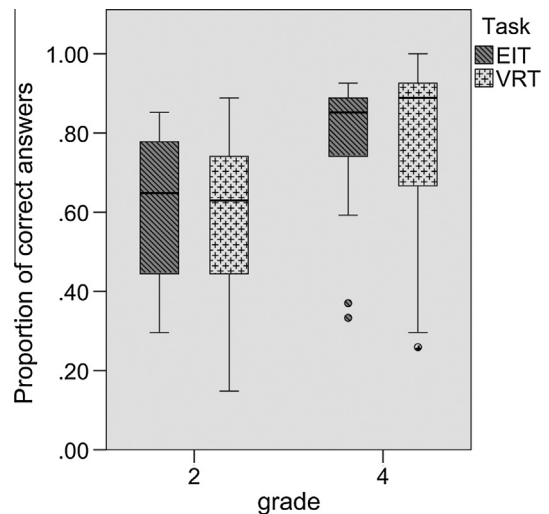
Fig. 7. Performance in Visual Recursion Task (VRT) and Embedded Iteration Task (EIT), across grades. Fourth graders had higher scores than second graders, in both VRT and EIT. Within each grade, the difference between tasks was not significant.

26.9% of the second graders had so. This difference was also significant ($\chi2 = 9.43$, $p = 0.002$). One child in the fourth grade and one in the second grade had performance scores lower than predicted by chance (i.e. equal or lower than 26%). This means that these children discriminated recursive items from foils more than 74% of the trials, but still consistently chose the foils. These two participants were excluded from further regression and correlation analyses involving VRT because even though they induced a rule that allowed them to distinguish recursive items from foils, they would be treated as performing worse than other participants performing randomly. Since we were interested in investigating the cognitive underpinnings of the ability to represent recursion, these two subjects would be ambiguous and noisy data points.[2]

Visual strategies. A central issue concerning our method is the question of whether participants were able to represent the structural self-similarity present in the recursive images; and to apply this knowledge throughout different VRT trials. One possible alternative to the representation of self-similarity would be the usage of heuristic strategies, based on the detection of simple salient features within the foils, which would allow their exclusion without an understanding of the underlying structure. In order to prevent the emergence of a systematic 'choice-by-exclusion' strategy, we used different categories of foils. Our assumption was that, if individuals were able to represent self-similarity, they would perform adequately in all different foil categories.

At the group level, the number of correct choices was significantly above chance for all foil categories and for both grade groups (Binomial test, $p < 0.005$). For detailed

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[2] However, we repeated all main analyses including these two participants and found a similar pattern of results.

analyses comparing performance across categories see Appendix C.

Visual complexity. Another important issue concerns the role of visual complexity. It is possible that the ability to perform adequately in VRT is limited by the capacity to cope with the amount of visual information. In our experiment, fractals of 'complexity 5' contained a higher number of elements (for instance, squares) than stimuli of 'complexity 3' (Fig. 5), and greater amount of visual information may be harder to process. To analyze this effect we compared the performance between trials displaying different amounts of visual complexity using a GEE with 'grade' as a between-subjects factor, and 'visual complexity' as a within-subjects factor. We found that visual complexity had a significant main effect on VRT performance (*Wald* $\chi 2 = 6.5$, $p = 0.039$). Specifically, the proportion of correct answers in the category 'complexity4' was higher than in the category 'complexity5' (*estimated marginal mean* (*EMM*) *difference* = 0.06, $p = 0.026$). All p-values were corrected using sequential Bonferroni correction. Detailed grade * visual complexity interaction analyses and figures are presented in Appendix D. Overall, higher levels of visual complexity yielded worse results, especially within second graders.

3.2. EIT

General overview: correct responses by grade. On average, children attending the fourth grade ($M = 0.78$, $SD = 0.18$) had a higher proportion of correct responses in EIT than children attending the second grade ($M = 0.62$, $SD = 0.17$). This was a significant difference (*Mann–Whitney U: z = −3.70*, $p < 0.001$; Fig. 7). While 77% of fourth graders had a proportion of correct answers above chance, only 35% of the second graders had so. This difference was also significant ($\chi 2 = 5.2$, $p = 0.023$).

Visual strategies. We repeated the analysis described for VRT, now with the proportion of correct answers in EIT as the dependent variable. Our results suggest that, at the group level, second graders performed randomly in the foil category 'odd constituent' (Proportion = 0.52, Binomial test, $p = 0.556$). For all other foil categories and for both grade groups, performance was significantly above chance (Binomial test, $p < 0.005$). Detailed comparisons across categories are presented in Appendix C.

Visual complexity. We repeated the complexity analysis described for VRT, with the proportion of correct answers in EIT as the dependent variable. We again found that visual complexity had a significant main effect on performance (*Wald* $\chi 2 = 12.6$, $p = 0.002$): The proportion of correct answers in the category 'complexity3' was higher than in the categories 'complexity4' (*EMM difference* = 0.06, $p = 0.012$) and 'complexity5' (*EMM difference* = 0.07, $p = 0.06$). All p-values were corrected using sequential Bonferroni correction. Detailed figures, interaction analyses, and subsequent pair-wise comparisons are presented in Appendix D. Overall, results suggest that visual complexity also plays a role in the ability to perform adequately in EIT, with fewer constituents easier to process

3.3. VRT vs. EIT and effects of task order

In order to compare children's performance in VRT and EIT, we ran a GEE model with 'grade' as a between-subjects factor, and 'task' as a within-subjects factor. There was a significant main effect of grade (*Wald* $\chi 2 = 12.9$, $p < 0.001$), but no difference between tasks ($p = 0.9$) and no interaction between grade and task (*Wald* $\chi 2 = 1.4$, $p = 0.24$), suggesting the grade effects were not specific to recursion (Fig. 7).

To assure the validity of comparisons between VRT and EIT, we balanced the order of the tasks in the procedure. However, we noticed that one of the 'task-order' conditions yielded lower performance than the other. Specifically, participants starting the procedure with VRT had a significantly lower response accuracy (on both tasks VRT and EIT combined; $M = 0.63$, $SD = 0.21$) than participants that started with EIT ($M = 0.72$, $SD = 0.17$; *Mann Whitney U* = 851, $z = −3.2$, $p = 0.001$). To further explore this, we first investigated whether performance was differently affected in different tasks and in different grades (Fig. 8).

Before testing the effect of task-order, and to better interpret potential interactions between 'task-order' ('VRT-EIT' vs. 'EIT-VRT') and 'task' (VRT vs. EIT), we recoded the former variable on a trial-by-trial basis. The new variable, called 'position', can be understood as the position of the task in the procedure. For instance, in trials where the task is 'VRT' and the order of tasks is 'VRT-EIT', the 'position' variable is coded as 'FIRST'. Likewise, in trials where the task is 'EIT' and the order of tasks is 'EIT-VRT', the 'position' variable is coded as 'FIRST', etc.

We ran a GEE model with 'task' (VRT vs. EIT) and position (FIRST vs. SECOND) as within-subjects effects, and 'grade' (second vs. fourth) as a between-subjects variable. We analyzed 'task', 'grade' and 'position' main effects, and all possible interactions. The summary of the model is depicted in Table 1.

We found significant main effects of 'position' and 'grade' on performance ($p < 0.001$), in agreement with the previous analyses. Furthermore, we found a significant interaction between 'task' and 'position'. Performance in EIT-FIRST position was better than performance in VRT-FIRST position (*EMM difference* = 0.15, $p = 0.004$). Conversely, VRT-SECOND position yielded better performance than EIT-SECOND position (*EMM difference* = 0.17, $p = 0.001$).

Within VRT, the proportion of correct answers was higher when this task was performed in the SECOND position of the procedure than when the same task was performed in the first position (*EMM difference* = 0.21, $p < 0.001$). Within EIT, there was also a trend towards higher accuracy when this task was performed in the FIRST position than when it was performed in the second position (*EMM difference* = 0.11, $p = 0.052$). All p-values were corrected with sequential Bonferroni.

Additional interaction analyses are presented in Appendix E.

Overall, results suggest that the order of the task in the procedure had a strong influence on task performance. Specifically, VRT accuracy is increased by previous experience with EIT. However, this effect of task-order was not
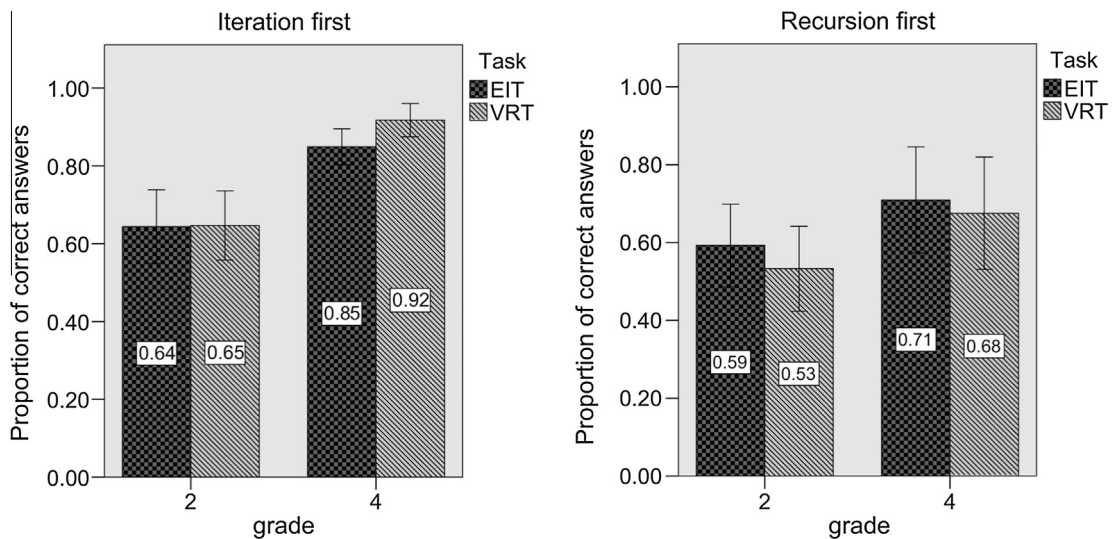
Fig. 8. Performance across different task-sequence conditions. In the sequence condition 'VRT-EIT' (right) participants performed the Visual Recursion Task (VRT) first; in the condition 'EIT-VRT' (left) participants performed the Embedded Iteration Task (EIT) first. Children who performed the iterative task first scored globally better than those who started with recursion. Crucially, starting the procedure with VRT decreased EIT accuracy. This suggests that children transferred knowledge from simple iteration to recursion, but not the other way around.

Table 1

Effects of task, position, grade and all interactions in the processing of visual hierarchies. Here we present the results of a General Estimating Equations Model with the 'correctness' (correct/incorrect) of each trial from Visual Recursion Task (VRT) and Embedded Iteration Task (EIT) as the dependent variable. Overall, fourth graders scored better in both tasks, and accuracy was better in the second task of the procedure than in the first task. However, there was a strong interaction between task and position: Performance in VRT was higher when this task was performed after EIT (i.e. in the second position of the procedure), but not vice versa (see Fig. 6). This suggests that children transfer knowledge from simple iteration to recursion, but not the other way around. We conclude that the overall similarity in accuracy between EIT and VRT masks interesting differences in learning constraints. QICC (Corrected Quasi Likelihood under Independence Model Criterion). The asterisk (*) denotes interaction.

Model ($QICC$ = 3252)	Type III	
	Wald $\chi 2$	p
Intercept	87.8	0.000
Task (VRT vs. EIT)	0.5	0.464
Position (First vs. Second)	7.7	0.005
Grade (Second vs. Fourth)	25.9	0.000
Task * Position	16.2	0.000
Task * Grade	3.6	0.057
Position * Grade	2.2	0.138
Task * Position * Grade	5.2	0.022

due to a practice effect during the experiment, since EIT performance decreased when this task was performed in the second position of the procedure.

3.4. Role of grammar comprehension ability and non-verbal intelligence

To assess whether the ability to represent visual recursion was predicted by language abilities, we tested all participants in the TROG-D, a test of grammar comprehension. Furthermore, to assess whether the potential effect of

grammar comprehension was independent of general capacity factors, we tested the same participants in a non-verbal intelligence task – The Raven's coloured progressive matrices (CPM). Participants' raw score in TROG-D was $M = 16.9$, $SD = 2.0$ (*minimum:* 13, *maximum:* 20), while CPM raw score was $M = 29.2$, $SD = 3.6$ (*minimum:* 21, *maximum:* 34). Segregated by grade group, results were the following: Second graders' score in TROG-D was $M = 15.9$, $SD = 2.0$ (*minimum:* 13, *maximum:* 20), while CPM raw score was $M = 27.9$, $SD = 3.6$ (*minimum:* 21, *maximum:* 34); Fourth graders' score in TROG-D was $M = 18.0$, $SD = 1.4$ (*minimum:* 16, *maximum:* 20), while CPM raw score was $M = 30.5$, $SD = 3.0$ (*minimum:* 23, *maximum:* 34). Overall, fourth graders scored significantly higher than second graders in both TROG-D ($t(50) = -4.5$, $p < 0.001$) and CPM ($t(50) = -2.9$, $p = 0.006$).

The overall proportion of correct answers in VRT was positively correlated with both CPM ($\rho(50) = 0.52$, $p < 0.001$) and TROG-D ($\rho(50) = 0.43$, $p = 0.002$) scores. Likewise, the proportion of correct answers in EIT was positively correlated with both CPM ($\rho(50) = 0.58$, $p < 0.001$) and TROG-D ($\rho(50) = 0.41$, $p = 0.003$) scores. To test whether grammar comprehension effects were specific to VRT and independent of general intelligence, we ran a GEE model with 'task' (VRT vs. EIT) as the within-subjects factors, and TROG-D and CPM scores as covariates. The summary of the model is depicted in Table 2. Our results suggest that grammar comprehension predicts performance of both VRT and EIT (main effect of TROG-D: *Wald* $\chi 2 = 6.7$, $p = 0.01$), and that this effect is partially independent from non-verbal intelligence since both main effects are significant. However these effects were neither specific for VRT nor for EIT (no interaction between task and TROG-D: $p = 0.54$). We repeated this analysis using the more specific variable 'embedded clauses' (number of TROG-D blocks containing embedded clauses which were answered

Table 2

Grammar comprehension is an independent predictor of visual hierarchical processing, but not specific of recursion. Here we present the results of a General Estimating Equations Model with the 'correctness' (correct/incorrect) of each trial from Visual Recursion Task (VRT) and Embedded Iteration Task (EIT) as the dependent variable. Grammar comprehension predicts performance in VRT, even after accounting for the variability explained by general intelligence. However, this effect is not specific to VRT but general to both VRT and EIT. QICC (Corrected Quasi Likelihood under Independence Model Criterion). The asterisk (*) denotes interaction.

Model (*QICC* = 3093)	Type III	
	Wald $\chi2$	*p*
Intercept	25.9	0.000
Task (VRT vs. EIT)	0.3	0.600
TROG-D (grammar)	6.7	0.010
CPM (intelligence)	22.3	0.000
Task * TROG-D	0.4	0.542
Task * CPM	0.0	0.971

correctly; maximum score = 5). The results were similar: There was a main effect of 'embedded clauses' (*Wald* $\chi2 = 5.4$, $p = 0.02$), independent of intelligence, but not specific to VRT (interaction task * embedded clauses: $p = 0.9$).

Finally, we analyzed the effects of grammar and intelligence within each grade group. We ran two GEE models, one for each grade (second and fourth). We found that CPM score (intelligence) was a predictor of both VRT and EIT within the second grade (*Wald* $\chi2 = 10.1$, $p = 0.001$), and fourth grade (*Wald* $\chi2 = 4.9$, $p = 0.03$); and that TROG-D score (grammar comprehension) was not an independent predictor of VRT and EIT performance within each grade group ($p > 0.1$), i.e. only CPM predicted performance within each grade group.

Importantly, CPM (intelligence) and grammar comprehension were not significantly correlated ($r = 0.25$, $p = 0.09$). Furthermore, partial correlations controlling for general intelligence (including all subjects of both grades) revealed that grammar comprehension was still correlated with both EIT ($r = .36$, $p = 0.01$) and VRT ($r = .32$, $p = 0.02$). Taken together these results suggest that a between-grade maturational factor is driving the correlation between grammar comprehension and both VRT and EIT, and that this effect is not completely explained by a general development in cognitive capacity. We will discuss the implications of these results in the next sections.

4. Discussion

In this study, we investigated for the first time the ability of children to represent structural self-similarity in visuo-spatial hierarchies. In this experiment we used visual fractals, which children are very rarely exposed to. Hence, we could investigate the ability to acquire novel recursive representations. Here, we aimed at investigating not only whether the ability to acquire recursive rules in vision followed a development course somehow similar to language, but also whether the acquisition of recursion in vision was constrained by similar factors as the acquisition of recursion in language. For this purpose we explored

the individual variation in visual processing efficiency, grammar comprehension and general intelligence.

We found that: (A) the majority of fourth graders performed adequately in both recursive and iterative tasks, while many second graders failed in both; (B) higher degrees of visual complexity reduced the ability to instantiate either recursive and iterative rules, but specially among the second graders; (C) recursive representations of hierarchical structures yielded better results than iterative representations in the detection of errors nested within lower visual scales; (D) there was an unexpected task-order effect: performance in visual recursion improved with previous experience with non-recursive iteration, but not vice versa; (E) both general grammatical abilities and first-order clause embedding were independent predictors of accuracy in the visual tasks, independently of the effects of non-verbal intelligence. However, this effect was general to hierarchical processing, and not specific to recursion. This means that even though CPM results (non-verbal intelligence) were predictive of visual recursion and iteration, there was a specific correlation between VRT, EIT and grammar comprehension, which was not explained by general intelligence. This could be an indicator of shared cognitive resources between language and vision in the processing of hierarchical structures.

Taken together, these results suggest that the ability to represent recursion and iteration may become available at similar stages during the ontogenetic development (around 9 years old). However, once this potential is present, other factors related with cumulative exposure to hierarchical structures may play a role in the representation of hierarchical self-similarity. For instance, in our study, prior experience with iterative rules was fundamental to the understanding of recursion (but not vice versa). These results mimic the findings of language research (Roeper, 2011). Our results also suggest that age differences can be partially explained by differences in visual processing efficiency, since the effects of visual complexity are more pronounced in second graders, and this group is especially impaired in the detection of 'odd' foils. Finally, also grammar comprehension abilities partially account for these grade differences, independently of general intelligence. This suggests that the ability to process hierarchical structures in the linguistic and visual domains partially recruit similar cognitive resources, although these resources are not specific to recursion. If recursion were central to all syntactic processes in language, we would expect to find a specific correlation between visual and linguistic recursion, instead of a general correlation with hierarchical processing. Thus, our results seem to challenge Chomsky's thesis (Chomsky, 2010).

4.1. Performance across grade

Our first important result was a demonstration that 9- to 10-year-old children are well able to represent recursion in the visual domain. The fact that they are able to do so without instructions or response feedback, and with only a very short training session (4 trials), suggests that they are spontaneously able to generalize the knowledge

of structural self-similarity across test items. Furthermore, we used different categories of foils, and found no performance differences between them. This suggests that children who passed VRT did not rely on simple heuristic strategies, and were probably able to perceive all features necessary to represent hierarchical self-similarity. The fourth graders were also able to correctly continue non-recursive iteration and there were no significant differences between recursive and non-recursive tasks, although more fourth graders tended to perform above chance in EIT than in VRT (77% vs. 69%).

Perhaps more surprising was the finding that many second graders performed poorly in both recursive and non-recursive tasks. Since second graders are able to handle conjunctions (e.g. "John, Bill, Fred, and Susan arrived.") and to some extent syntactic structures like "What is the color of Bill's dog's balloon?" (Roeper, 2007; Roeper, 2011), we might expect them to perform adequately in a visual task that requires the representation of iterative processes embedded within hierarchical structures. However, only 35% of second graders scored above chance in EIT (and only 27% performed adequately in VRT). There are several possible interpretations for these results: On the one hand, it is possible that the ability to represent iterative processes and hierarchical structures in the visual domain is not within the cognitive repertoire of second graders. On the other hand, it is possible that even though the potential to represent these structures is available, other factors related to our particular instantiations of iteration (or recursion) impaired their ability to make explicit judgements. One such factor might be the amount of visual complexity. Another factor may be that these children likely had little or no previous experience with visuo-spatial fractals before performing our experiment.

4.2. Effects of visual complexity

Overall, we found that higher levels of visual complexity reduced participants' ability to extract recursive and iterative principles. This effect seems to be more pronounced in the second grade group. Incidentally, we asked the majority of children (18 second graders and 24 fourth graders) how frequently they had detected differences between the choice images during the realization of our tasks (i.e. between foil and correct fourth iteration). While 17.6% of the questioned second graders reported perceiving no differences between 'correct' fourth iteration and foil most of the time, only 4.5% of the fourth graders did so. This provides additional evidence that younger children may have had difficulties detecting (or retrieving) information relevant to process the test stimuli. Previous research on the development of hierarchical processing suggests that before the age of 9 children seem to have a strong bias to focus on local visual information (Harrison & Stiles, 2009; Poirel et al., 2008), which as we have discussed, can affect normal hierarchical processing. Thus, further research will be necessary to determine whether the potential to represent recursion in vision is not part of the cognitive repertoire of many younger children; or whether inadequate performance was caused by inefficient visual processing mechanisms.

4.3. Dissociations between VRT and EIT: 'Odd foil' detection and task-order

Although we found no significant performance differences between VRT and EIT in overall, a closer analysis revealed two interesting dissociations:

First, unlike in VRT, children seemed to have difficulty in rejecting the 'Odd constituent' foils in EIT, though performance was adequate in trials containing other foils categories ('Positional error' and 'Repetition'). Since they were able to respond adequately to this foil category while executing VRT, it seems unlikely that this result was caused by a general inability to perceive 'odd constituent' mistakes. Instead, we suspect that there may be differences in the way recursive and non-recursive representations are cognitively implemented. These differences might have led subjects to detect errors of the 'odd constituent' type more efficiently in VRT. Previous studies (Martins & Fitch, 2012) suggest that EIT may be more demanding of visual processing resources than VRT. Moreover, we found here that the effects of visual complexity in EIT were broader than in VRT, extending not only to the second grade, but also to the fourth grade (see Appendix D). If performance in EIT is more dependent on bottom-up perceptual resources, and more sensitive to variations in low-level visual information, then it is plausible that subtle errors are harder to detect in this task than in VRT. In the 'Odd constituent' foils, these errors occur deeply nested within the hierarchical structure (i.e. at the smallest size scale), and only in a subset of hierarchical nodes. Elsewhere, it has been argued that recursive representations may be more efficient than non-recursive representations at encoding of hierarchical structures (Koike & Yoshihara, 1993; Martins, 2012). This greater efficiency might derive from the fact that the same "rules" can be used to represent different hierarchical levels, hence allowing a simultaneous encoding of the whole and of the details. Particularly in the visual domain, there is evidence that compressed representations lead to a better perception of fine-grained details (Alvarez, 2011).

A second difference found between VRT and EIT was the effect of task-order. Previous experience with EIT seemed to help children to perform adequately in VRT. However, the inverse effect was not found, i.e. previous exposure to VRT did not enhance EIT accuracy. This asymmetry suggests that VRT performance enhancement after EIT was not due to a general learning effect. Instead, we think that this finding reflects different characteristics of recursive and iterative representations.

As exemplified in Fig. 1, recursion is a particular subset of hierarchical embedding, where both elements of a transformation rule are perceived as belonging to the same category. It seems possible that children may require exposure to simpler iterative processes before they are able to identify hierarchical self-similarity. The reason why recursion may be harder to acquire could be related to the fact that constituents within recursive representations are at a higher level of abstraction. For instance, in our EIT stimuli (Fig. 3), it suffices to build a representation of the initial structure [B], and of the constituents [C] being added into that structure: 1. [B]; 2. [B[C]]; 3. [B[CC]]; 4. [B[CCC]]. In recursion, in order to predict the next iteration,

participants are required to encode successive hierarchical levels with the same rules. This requires the formation of an abstract category [A], which incorporates the features of both [B] and [C] (Fig. 3). In order to generate a representation of [A] and [A[AAA]], previous experience with [B] and [C] may be required. This explanation is consistent with the previous findings on language recursion (Roeper, 2011), and lends further support to the alternative hypothesis that biological maturational factors are not the main factor limiting the ability to represent recursion, once the ability to represent iteration is available.

4.4. Visual recursion and grammar

A final hypothesis tested in our study was that grammar comprehension and visual recursion would be correlated. We found that the ability to represent recursion in the visual domain was correlated with grammar comprehension, and that this correlation was partially independent from general intelligence. However this effect was not specific to recursion, since grammar comprehension also correlated with embedded iteration. This suggests that grammar comprehension abilities were correlated with a more general ability to represent and process hierarchical structures generated iteratively, independently of whether these were recursive or not. This result is not completely surprising given that not all syntactic structures in TROG-D are recursive, although all are hierarchical.

We also assessed whether there was a more specific correlation between visual recursion and embedded clauses, but found again only a general association with both EIT and VRT. However, it is important to note that TROG-D only includes sentences with one level of embedding, e.g. relative clause (nominative): *Der Junge*, **der** *das Pferd jagt*, *ist dick* 'The boy, who is chasing the horse, is chubby'. Children may potentially use non-recursive representations for these kind of sentences (Roeper, 2011). Only a task focussed on sentences with several levels of recursive embedding would allow a direct comparison between visual recursion and syntactic recursion. Despite this limitation, it is interesting that performance on our novel visual tasks was correlated with grammar abilities, even when the effects of non-verbal intelligence were taken into account. These correlations could be explained by the existence of shared cognitive resources, independent from non-verbal intelligence, used for the processing of hierarchical structures in both language and visuo-spatial reasoning, or even by the effects of literacy (which are partially independent of intelligence) in the processing of hierarchical structures. Interestingly, while individual differences in intelligence predicted VRT and EIT scores both between and within grades, grammatical comprehension abilities accounted only for differences between grades. Again, this argues in favor of a general age-related maturational influencing the processing of hierarchical structures, occurring between second and fourth grade, which is partially independent from non-verbal intelligence. Furthermore, in our sample, grammar

comprehension and non-verbal intelligence were not significantly correlated. Hence, this general maturation process in hierarchical processing cannot be explained solely by the increase of intelligence with age.

Future studies with a more comprehensive assessment of grammar (that includes recursion at several levels), and the inclusion of more cognitive tests (assessing cognitive control, attention, etc.) in the experimental procedure could potentially shed more light on a possible relationship between grammar and processing of complex visual structures.

5. Conclusion

In this study we assessed for the first time the ability of children to represent hierarchical self-similarity in an unambiguously non-linguistic domain. Consistently with previous findings on language (Miller et al., 1970) and visual-spatial research (Harrison & Stiles, 2009; Poirel et al., 2008), we found that the majority of fourth graders, but not second graders, were able to adequately process visual fractals generated using both recursive and iterative rules. This difference is partially accounted by distinct visual processing efficiency levels, but it is also predicted by grammar comprehension. Two crucial differences seem to emerge between the representation of recursive and iterative processes: (1) While the ability to acquire recursion seems to be facilitated by previous learning of non-recursive representations, the opposite is not true; (2) Though recursive representations are harder to learn, once acquired, they seem to enhance the processing of hierarchical details.

In sum, we have found an interesting developmental path in the ability to represent hierarchy and recursion in the visuo-spatial domain. This path might be influenced by biological (maturational) factors, and by the exposure to particular kinds of stimuli. On the one hand, the re-organization of brain networks (Power et al., 2010), for instance, the myelination of the superior longitudinal fasciculus (occurring around the ages 7–8), seems to increase the efficiency of hierarchical processing (Friederici, 2009); on the other hand, the acquisition of certain hierarchical categories might depend on a gradual exposure, from concrete to abstract, where knowledge builds up incrementally (Dickinson, 1987; Roeper, 2011; Tomasello, 2003). Children may be born with a latent innate ability to detect and represent hierarchical structures (Berwick et al., 2011), but the development and precise tuning of this ability may require experience with enough examples to allow inductive generalizations (Dewar & Xu, 2010) and to allow acquisition of domain-specific constraints (Perfors et al., 2011a; Perfors et al., 2011b). Although the developmental time course of recursion in language and vision seem to obey similar constraints, this study does not provide direct evidence that the same cognitive machinery is used in both domains. However, it does provide a crucial method and important results, which offer a clear path for further investigation on the interface between language and visual aspects of cognition.

Acknowledgments

Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.cognition.2014.05.010.

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