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Review

Distinctive signatures of recursion

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Although recursion has been hypothesized to be a necessary capacity for the evolution of language, the multiplicity of definitions being used has undermined the broader interpretation of empirical results. I propose that only a definition focused on representational abilities allows the prediction of specific behavioural traits that enable us to distinguish recursion from non-recursive iteration and from hierarchical embedding: only subjects able to represent recursion, i.e. to represent different hierarchical dependencies (related by parenthood) with the same set of rules, are able to generalize and produce new levels of embedding beyond those specified *a priori* (in the algorithm or in the input). The ability to use such representations may be advantageous in several domains: action sequencing, problem-solving, spatial navigation, social navigation and for the emergence of conventionalized communication systems. The ability to represent contiguous hierarchical levels with the same rules may lead subjects to expect unknown levels and constituents to behave similarly, and this prior knowledge may bias learning positively. Finally, a new paradigm to test for recursion is presented. Preliminary results suggest that the ability to represent recursion in the spatial domain recruits both visual and verbal resources. Implications regarding language evolution are discussed.

Keywords: recursion; hierarchy; embedding; representation; language

1. INTRODUCTION

Recursion is one of the most controversially discussed terms in the cognitive sciences. Although it has been hypothesized as a human-unique trait and as a necessary capacity for the evolution of language [1], the multiplicity of definitions being used [2–6] has undermined the broader interpretation of empirical results [7]. One of the major problems that stems from this multiplicity is the difficulty in drawing boundaries between recursion and similar processes such as cognitive grouping, hierarchical embedding and iteration [8].

Although there has been a proliferation of literature arguing for and against the claim of recursion as uniquely human [2,8–14], the debate remains unresolved. Some empirical paradigms have been considered relevant to address the topic [15–18] but because they fail to capture the distinction between recursion and hierarchical embedding, over-interpreting the results may be misleading [12,19].

Given that brain computations are opaque to observers (until behavioural correlates have been found), definitions focused on algorithmic properties (such as 'a recursive function is one that calls itself') may not be entirely relevant for empirical research. On the other hand, isolated analyses of signals (such as vocalizations, social interactions, etc.) may be misleading because not all structures that can modelled using recursion are produced by recursive processes, neither are these structural properties necessarily perceived by observers.

To overcome these difficulties, I propose that only a definition focused on representational abilities such as 'the ability to represent self-similarity across hierarchical levels' enables the prediction of recursion-specific behavioural traits: if a subject is able to represent different hierarchical levels, i.e. different hierarchical nodes related by 'parenthood' (in the graph theory sense), with the same set of rules, then he or she may be able to generalize and generate new levels of embedding ('child' nodes) beyond those specified *a priori* (whether in the algorithm or in the input).

Defined as such, recursion may provide advantages to its users in the domains that it is available: it may provide prior knowledge regarding new or unknown hierarchical levels; and if shared by a population, then it may contribute to the establishment of communicative conventions. Here, it is important to make explicit that we analyse recursion as a kind of representational abstraction without considering how it could be implemented in the brain. Recursion could be a single module recruited by different modalities or it can be an umbrella term referring to a set of mechanisms that operate independently in different domains, each with its own specific constraints. The empirical research essential to support any of these hypotheses has been delayed by the shortage of tools to assess the use of recursion in non-linguistic domains.

Under this framework, a new paradigm to test for recursion in the visuo-spatial domain will be presented. Given that it can be applied independently of language and in a non-serialized modality, it has the potential to

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One contribution of 13 to a Theme Issue 'Pattern perception and computational complexity'.

provide insights into the relationship between recursion and language in the evolutionary history.

2. RECURSION: FROM OPERATION TO STRUCTURE

As pointed out by Fitch [7], recursion has been many things to many people. Some definitions focus on the characterization of recursive computations; others attempt to describe which structures can be considered recursive.

In modern computer science, a recursive function is one that calls itself, or one that is defined in terms of itself [7,10]. However, in logics, 'recursive' can mean 'computable' (i.e. if membership of the function products can be determined by a Turing machine) [3,20], or refer to the process of defining something in terms of something previously defined [2,4–6]. As pointed out by some authors [7,8], this latter definition is too broad because it includes computations that specify items in terms of simpler items and therefore any operation able to generate hierarchies (as occurs in cognitive grouping and in different perceptual domains [11]). In the most restrictive sense of recursion ('specific recursion'), the items being combined (or embedded) should be categorized as of the same kind as the ones they generate (or are embedded on) [7,8,10].

Although definitions focused on the process can be a good start to define which phenomena we are trying to grasp, they are not completely useful for empirical purposes. Because the implementation of a computation is opaque to the observer (at least before some behavioural correlate has been found), a better empirical approach is to search for distinctive signatures in the output that may suggest the presence of that computation. In the case of recursion, those signatures are usually the presence of structural self-similarity or the embedding of constituents within constituents of the same kind [7,10].

3. RECURSION: FROM STRUCTURE TO REPRESENTATION

Recursive structures (in the strict sense) are ubiquitous in human activity and have been claimed in visual art [21], music [22,23], architecture [24], humour [25], second-order theory of mind [26], problem-solving [27], action sequencing [28], syntax [29–31], prosody [8,32] and conceptual structure [33,34]. These cultural achievements are present not only in modern societies but also in pre-industrial and ancient civilizations.

In spite of the pervasiveness of structures that can be modelled using recursive algorithms or rule sets, not all of them will be represented as such. This means that the amount of recursion in a structure will only be relevant for an observer to the extent that he can decode it meaningfully. For example, in the Kotoko architecture [24], self-similarity in different scales is built consciously, subjected to abstract representation and used to convey a meaning (e.g. social ranking). In such circumstances, we can say that both producers and observers have the ability to represent the underlying recursive structure. On the other hand, although we can model the long-distance tensional structure (e.g. tonal deviation from the tonic) in music as recursive, untrained listeners may not be sensitive to such properties [22,23]. Likewise, even if we can use recursion to model baboons' social hierarchies (J. Flack, R. Jackendoff, D. C. Krakauer & S. A. DeDeo 2011, personal communication), this does not imply that baboons—in spite of their success in social navigation [35]—are able to represent recursion. In the latter example, it is possible that individuals use separate rules to represent different hierarchical levels ([X is dominant over Y]; [Y is dominant over Z]) instead of using recursive rules to encode dominance ([X is dominant over Y [who is dominant over Z]]) [36].

The opacity of algorithmic processes can be further exemplified by one of the first structures described using a recursive generating rule: the Fibonacci sequence. In 1201, Leonardo de Pisa described a sequence of numbers where each member of the sequence $S_{(n)}$ could be obtained by the sum of the two previous members: $S_{(n)} = S_{(n-1)} + S_{(n-2)}$. Although the Fibonacci sequence (1 1 2 3 5 8 13, etc.) can be implemented using a recursive algorithm, with a function that calls itself:

```
def fib(n):
  if n = 1 or n = 2: return 1
  return fib(n − 2) + fib(n − 1),
```

it can also be implemented with a non-recursive simple iterative loop:

```
def fib_iter(n):
  if n = 1 or n = 2: return 1
  pre = 1
  prepre = 1
  for i in range(3,n):
    pre, prepre = pre + prepre, pre
    return pre + prepre.
```

An isolated analysis of the output/signal is insufficient to determine the underlying computation, therefore the fact that a certain individual can produce the Fibonacci sequence tells us little about his ability to use or represent recursion. Independently of how the sequence is produced, if a given observer is able to use recursion to represent the subset that he receives from the input, then he or she may display specific behaviours while generating further elements. In §4, we will discuss these distinctive behaviours in more detail.

In summary, not all activities that can be synthesized with recursive processes are be perceived as structurally meaningful by the observers. Hence, the ability to produce recursive structures and the ability to decode them do not necessarily come together [2,37]. Given that the ability to represent self-similarity in a structure (regarding a certain feature) may result in distinct behaviours, questions concerning representational abilities are more tractable empirically.

For the purposes of this paper, we define representation as a relationship between two objects (O1 and O2), where a given set of characteristics {C} of an object (O1) can be retrieved from another (O2). A cognitive representation entails that some change at the neural level (O2) occurred owing to the perception, storage or processing of certain features {C} present in the object (O1). That neural change (O2) will have a causal relationship to a secondary process (O3) that can

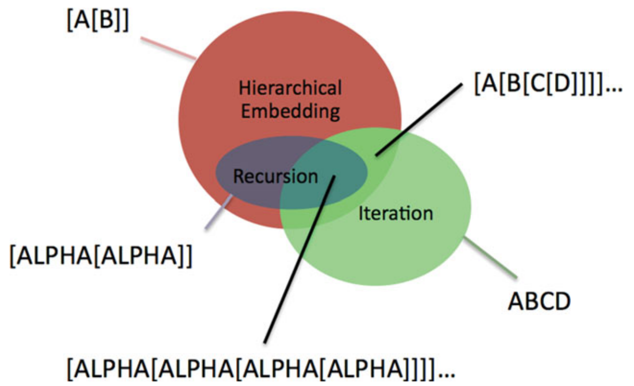

Figure 1. Examples of structures produced by iteration, hierarchical embedding and recursion, and by the combination of these processes. Constituents represented with the same letter are perceived as similar regarding a certain feature of relevance to the hierarchical structure. Brackets mean embedding. 'ALPHA' represents a general category that includes all letters in the alphabet.

be *measured* (for example, BOLD signal or behaviour). Under this assumption, one can detect whether a feature was represented while remaining agnostic regarding the nature or implementation of that representation.

Following this framework, in order to plan experiments and interpret behavioural responses, we first need to make theoretical distinctions between recursion and related processes such as non-recursive iteration and non-recursive embedding.

4. ITERATION, HIERARCHICAL EMBEDDING AND RECURSION

Iterative processes involve the repetition of an operation a given number of times. These processes may or may not generate hierarchical structures and may or not create dependency relationships between different elements.

Hierarchical structures involve the embedding of constituents within other constituents. If the embedding involves constituents of the same category it is called recursive embedding (in the strict sense), otherwise it is called non-recursive. Iteration, hierarchical embedding and recursion are not mutually exclusive. Nevertheless, it is possible to segregate the cognitive abilities that are necessary to represent the kind of information that each encodes (figure 1). In the subsequent sections, I will discuss how.

(a) *Iteration without embedding*

Consider a set composed by the ordered and indexed alphabet list. Call this set ALPHA. An iterative process example could be:

(1) Add the element ALPHA(n) to the structure $x(n)$ until $n = 3$. Each cycle add 1 to n.

$x0$ = A
$x1$ = AB
$x2$ = ABC
$x3$ = ABCD

In this process, each iterative step is a separate act that can exist independently from the others [30,36]. Such processes can create infinite sequences by unlimited concatenation [38] but can neither encode dependency relationships nor create new hierarchical levels [10] (J. Flack, R. Jackendoff, D. C. Krakauer & S. A. DeDeo 2011, personal communication). The encoding of dependencies requires the representation of rules that allow embedding, i.e. the representation that some constituents are dependent of other constituents, either structurally or functionally. Consider the next example:

(b) *Iteration with embedding*

(2) Implement one of the following rules. Repeat the cycle three times:

— embed B on A
— embed C on B
— embed D on C.

Again, well-formed structures could be:

$x0$ = [A]
$x1$ = [A[B]]
$x2$ = [A[B[C]]]
$x3$ = [A[B[C[D]]]]

For a given observer, the shape of the output in both examples, (1) and (2), could be similar: '*ABCD*'. However, if the observer is able to represent rules of embedding, then he has the possibility to interpret the positional attributes of the string '*ABCD*' as containing information about dependency relationships (e.g. 'right constituents are dependent over left constituents'). This perceptual decision requires access to semantic information and will be influenced by biases that can be innate, cultural or contextual (for example, prosodic cues can influence syntactic interpretations).

Considering the big picture of comparative cognition, there are three empirical questions relevant at this level: (i) Which species possess the ability to represent dependency relationships? (ii) In which domains are they able to do so? (iii) Which factors influence the perception of a structure as having dependency relationships?

(i) *Single versus multi-constituent hierarchical levels*

Iterative processes that allow embedding (such as (2)) can generate hierarchical structures. However, the same process that can create hierarchies with more than one constituent per level ([A[BB[C]]], [A[B[CC]]] or [A[BBB]]) can be used to create hierarchies with only one (non-empty) constituent per level (e.g. A[B[C[D]]]).

Potential differences in the processing of hierarchical nodes with one or several dependents relate to the ability to use memory to keep track of non-adjacent dependencies when the hierarchical information is presented linearly. If memory constraints are not an issue or if structures are presented nonlinearly, then the same representational abilities should allow the encoding of both single and multi-constituent hierarchical information. This means that the ability to process long-distance dependencies is not a specific signature of recursion, but general to hierarchical processing when there is more than one dependent per hierarchical node.

Consider the strings used in artificial grammar learning studies with the structure A^nB^n [16]: In $A_1A_2B_2B_1$, for example, the semantic content of the inner 'A_2B_2' is not modified by the outer 'A_1__B_1'. In fact, there is no real dependency relationship between the inner and outer 'AB', because they can exist independently and without changing the properties of each. The potential 'dependency' in these structures relates to the association between 'A_n' and 'B_n' and not between [AB] and another [AB]. Thus, and contrary to what has been argued [12,13], the ability to process A^nB^n structures (when embedded structures are not semantically related) cannot be considered as a specific trait of recursion [36]; neither are structures without long-distance dependencies necessarily deprived of recursion (such as in tail-recursion).

A more interesting issue relates to the limits of hierarchical processing with non-recursive embedding rules. If we consider the process (2) and the kind of structures it generates, we realize that each hierarchical level has to be represented individually. In these circumstances, we can embed an infinite number of constituents within the same hierarchical level [38], but we cannot create new hierarchical levels unless they are specified *a priori* (either as explicit rules in the algorithm itself, or acquired via the input). Recursion overcomes this limitation.

(c) *Recursive embedding*

Within the same framework, a recursive generation rule would be:

(3) Embed a member of the ALPHA set in another member of the ALPHA set.

Again, we can obtain structures such as

$x0 = [A]$
$x1 = [A[B]]$
$x2 = [A[B[C]]]$
$x3 = [A[B[C[D]]]]$

If a rule such as (3) is used, all elements of ALPHA are represented as having the same properties (relative to the fact that they belong to the same set, although these elements can differ in many other characteristics). Hence, the structure [A[B[C[D]]]] can be perceived as equivalent to [ALPHA[ALPHA[ALPHA[ALPHA[ALPHA]]]]]. Within this framework, new hierarchical levels can be represented without new rules being specified, as in the structure:

$x4 = [A[B[C[D[E]]]]]$

Moreover, with the same set of rules, we can represent new design features that might be useful for dynamic hierarchies:

- inversion of the previous order of dependency: [B[A[C]]]; and
- expression of bilateral dependency relationships: [A[B[C[B]]]], etc.

Obviously, such representations rules can be useless if unconstrained since they are too general. However, the availability of these rules to represent hierarchical structures may be advantageous in terms of flexibility [7], and might be the only practical method for large and highly complex hierarchies [39].

In spite of these processing advantages of recursion, it is not clear to what extent they are relevant empirically, given that it may be difficult to distinguish between an algorithm with a large set of rules and one that uses recursion [40]. For this reason, in my opinion, the key 'functionalist-cognitive accomplishment', as Harder [41] puts it, 'is the ability to take one incremental step beyond the given'. This means that the key empirical test for recursion is the ability to represent dependency relationships that were not previously defined, or to represent information within hierarchical levels not previously 'available'. What this ability presupposes is the knowledge (or expectation) that all nodes within a hierarchy can behave similarly and can display the same properties relatively to the way they interact with the nodes 'above' and 'below'. This allows, for example, that we embed a noun phrase inside a noun phrase already embedded in a noun phrase ([$NP_{(n-1)}$ [$NP_{(n)}$[$NP_{(n+1)}$]]]); or that each individual in a social hierarchy is represented has having both dominants and dependents. In §5, I will discuss how such properties might have provided evolutionary advantages.

The main point of this section is that different behavioural signatures may enable the detection of different cognitive processes:

- Iteration: ability to represent repetition of constituents.
- Hierarchical embedding: ability to represent dependency relationships between constituents.
- Recursive embedding: ability to represent new hierarchical levels (or new dependency relationships) beyond the given (innately or beyond the observable).

5. WHAT IS RECURSION GOOD FOR?

The ability to take steps beyond the given (regarding hierarchical embedding) and the ability to represent different hierarchical levels with the same set of rules may provide several advantages in the domains it is available:

(1) Within a hierarchical system, recursion allows the same way of thinking across different levels and the generation of new levels of embedding [27]. This entails the possibility of unbounded subdivision of each constituent into further subordinate constituents (useful in problem-solving [42], action sequencing [43–45], etc.), and the combination of elements creating new dominant constituents (e.g. the combination of primitive concepts into new concepts [33], an important feature of human creativity [34]). In this regard, it is important to note that these properties are distinct from cognitive grouping (often taken as a synonym of recursion [11,23,32]): although cognitive grouping may allow the clustering of existing constituents in supra-constituents (e.g. the organization of a visual array in clusters like 0000 → [00][00] → [[00][00]]), it does not allow the generation and recruitment of new constituents, for example, the subdivision of each '0' into '[00]', generating the structure [[[00][00]][[00][00]]].

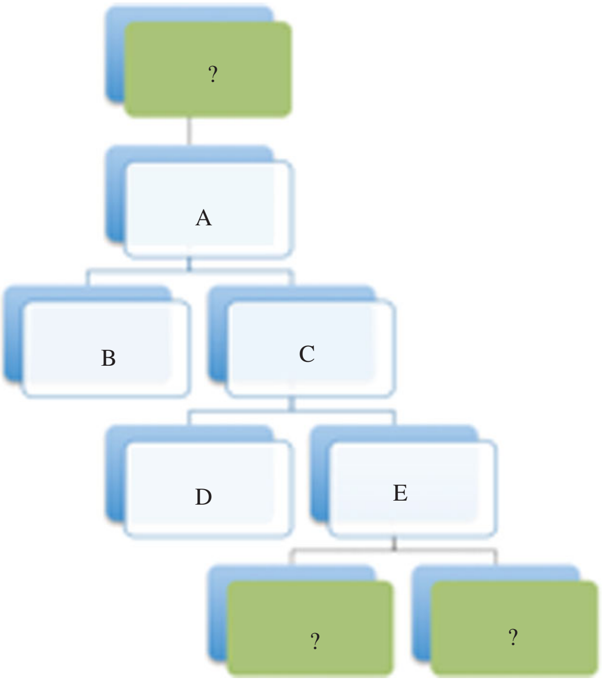

Figure 2. Expectation of self-similarity in a social hierarchy: by observing a series of interactions between the individuals (A, B, C and D), a given observer may infer the dominance relationships depicted in white boxes. If recursive rules are used to represent these relationships (e.g. 'each Y member of the hierarchy has two Y subordinates and one Y dominant'), then the observer might expect [E] to have subordinates and [A] to have dominants (green boxes).

(2) Recursion is an efficient method to encode complex hierarchies whether or not they were generated recursively [27,39,42]: if a given observer is able to build a compressed abstract representation of an entire hierarchy, then he can focus his attention on a small subset of constituents without losing track of the contextual ensemble. An eventual orthogonal and simultaneous representation of both the whole (abstract) and the details (perceptual) would constitute an efficient and accurate real-time strategy to parse complex hierarchical information. This kind of representation, already described for vision [46,47], could be useful if available in other domains, such as social and spatial navigation, where unseen (or unknown) landmarks have to be implicitly represented. Although this processing seems to occur in an automatic way, it is possible that in some domains the generation of complex abstract rules implies a slower and more cognitive demanding acquisition phase. Currently, it is unknown where such a phase is required and whether its processing is domain-specific.

(3) By allowing an implicit abstract representation of unseen or unknown constituents, the availability of recursion may decrease the amount of uncertainty regarding the interaction with hierarchical structures [14]. If an implicit representation would be generated automatically, this could bias individuals to perceive hierarchies as self-similar, and to expect similar behaviours at different hierarchical levels (figure 2). This predisposition could be advantageous to the extent that most hierarchies can be usefully modelled as self-similar; or to the extent that the expectation of self-similarity is better than no expectation.

Here, it is important to note two points: (i) if there are no priors regarding which rules are possible or likely to represent the output of a given system, then it may be impossible to generalize and predict its future behaviour [28,48]; (ii) although real objects deviate from abstractions (such as apples deviate from spheres), it seems that, at least in visual processing, abstract prototypes and the amount of deviation from those prototypes can be simultaneously represented in a mutually informative way [46]: the amount of deviation allowed before a shift from the initial abstraction is dependent on the suitability of the resulting behaviours [14,49]. Although recursive prototypes can be the initial priors, the representational schema may evolve owing to functional constraints. Updated rule sets can contain non-recursive hierarchical templates that are less powerful and more restrictive. Independently of the initial state of a given representational system, the final rule set can assume a variety of forms [40].

This hypothesis raises the empirical question of whether humans (or other species) try to collapse hierarchies into recursive prototypes (until the amount of perceived deviation forces cognitive set shifting).

(4) An eventual predisposition to interpret hierarchies as self-similar may be useful for the transmission of information and for the acquisition of communication systems: on the one hand, given that self-embedding processes can generate self-embedded structures (e.g. in prosody and syntax), the resulting isomorphism can increase the precision of decoding [1,37]; on the other hand, if such cognitive biases are shared by a population, then this would increase the likelihood of the emergence of a conventionalized system to transmit information [50,51]. Recent work has shown that the presence of recursive rules as Bayesian priors may enhance the acquisition of syntactic rules [52]. This seems to support the notion that the ability to represent recursion may bias learning positively.

(a) *Recursion and communication*

Much has been speculated about the relationship between recursion and language. However, in recent discussions, some convergence has started to form around the idea that recursion might have been available in other domains before the emergence of language [1,9–11,28,45], and that further modifications in human cognition made it available for communication [1,9–11].

A plausible candidate for this 'further modification' seems to relate to the hypothesis that the phonological output of private speech might help to serialize private cognition increasing the attention focus on one train of thought and strengthening the short-term memory capacity [7,10]. Another plausible candidate seems to be the ability to build symbolic representations [53] (not necessarily available in other serialized domains such as prosody and motor sequencing).

Although the processing of information, for example in the visuo-spatial domain, seems to occur independently from the ability to serialize thought, it is an open question whether the generation of recursive abstract rules can occur without serialization

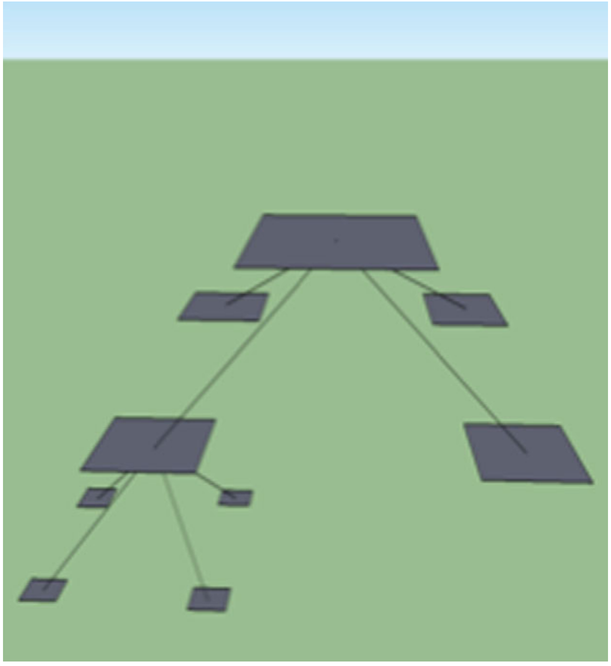

Figure 3. Recursive visuo-spatial hierarchy. A two-dimensional visual structure can be represented as three-dimensional hierarchy where bigger constituents are dominant over smaller constituents.

and/or without symbolic representation. If it seems to be true that principles of perceptual abstraction may be employed in non-linguistic domains [11,32,46], it also seems that the usage of language and serial representations in these modalities might enhance the processing accuracy [54].

The assessment of these hypotheses must be empirical, and could be systematized with the following questions:

- Are humans (or other species) biased to interpret hierarchies as self-similar (i.e. as structures where successive mother–child dependency relationships can be described by the same rules)?
- Are humans (or other species) able to represent new hierarchical levels (beyond the given)?
- Is this ability available in non-linguistic domains? If so, in which domains? and
- If available in other domains, is it dependent or enhanced by language (or by symbolic serialization)?

In §6, I will describe a new method that could be used to address these questions and report some preliminary results.

6. VISUAL RECURSION TASK

A new task has been developed [55] to assess the ability to represent visuo-spatial hierarchies as recursive structures (figure 3), and to apply these representations in the production of new levels of embedding. This method, called visual recursion task (VRT), is based on the properties of geometrical self-similar fractals, which can be generated by applying recursive embedding rules a given number of iterations.

In a typical VRT stimulus, subjects are exposed to the first three iterations of a fractal structure generation. Then they are asked to choose, from two possible alternatives, which correspond to the 'correct' answer (figure 4; correct in this sense means the fourth iteration of the generating process).

In theory, in order to correctly generalize a particular recursive rule to further iterations, subjects have to: (i) acquire categorical knowledge about constituents (shape and position), (ii) recognize that constituents are structured hierarchically (with dominance and subordination relationships); (iii) recognize that constituents at different hierarchical levels display similar positional properties (e.g. 'each triangle has smaller triangles at its vertices'); and (iv) apply the abstracted rule one level beyond the given.

To distinguish between recursion and embedded iteration, a non-recursive control task was also developed. In this task, iterative processes embed constituents within fixed hierarchical levels, without generating new levels (figure 5).

After validating the tasks, we tested different populations in both, together with some well-standardized cognitive measures of fluid intelligence and working memory (Weschler Abbreviated Scale of Intelligence matrix reasoning, digit span and corsi blocks). We found that

- The percentage of correct responses was lower in visual recursion than in embedding iteration (87% versus 92%) and the response time was longer (18 versus 16 s).
- Fluid intelligence was the best predictor of both visual recursion and embedded iteration (25% and 35% of the variance, respectively). However, while visual recursion accuracy was better predicted by verbal working memory than by spatial working memory; the opposite pattern was found in the embedded iteration task.
- Taken together, embedded iteration and the processing component of verbal working memory accounted for 60.4 per cent of visual recursion variance (embedded iteration: $\beta = 0.554$, $t = 4.367$, $p < 0.001$; verbal working memory: $\beta = 0.404$, $t = 3.184$, $p = 0.004$). Similarly, visual recursion and spatial working memory together predict 64.2 per cent of embedded iteration variance (visual recursion: $\beta = 0.588$, $t = 4.015$, $p = 0.001$; spatial working memory: $\beta = 0.378$, $t = 2580$, $p = 0.018$).

If we take working memory as a measure of the ability to store and manipulate information, then it is interesting that there is a dissociation in the modality of information processing that better predicts visual recursion and embedded iteration: (i) the ability to reverse the order of a given sequence of digits without losing track of the original sequence predicts accuracy in visual recursion even when all shared variance with embedded iteration is accounted for; (ii) on the other hand, the ability to reverse a visuo-spatial sequence predicts accuracy in embedded iteration even when all shared variance with visual recursion is accounted for. This may reflect the fact that recursive hierarchies are more regular [56], hence can be better represented by compressed abstract rules. Once these rules are used to encode information across hierarchical levels, this might reduce the visual memory load necessary to represent each constituent individually [39,46].

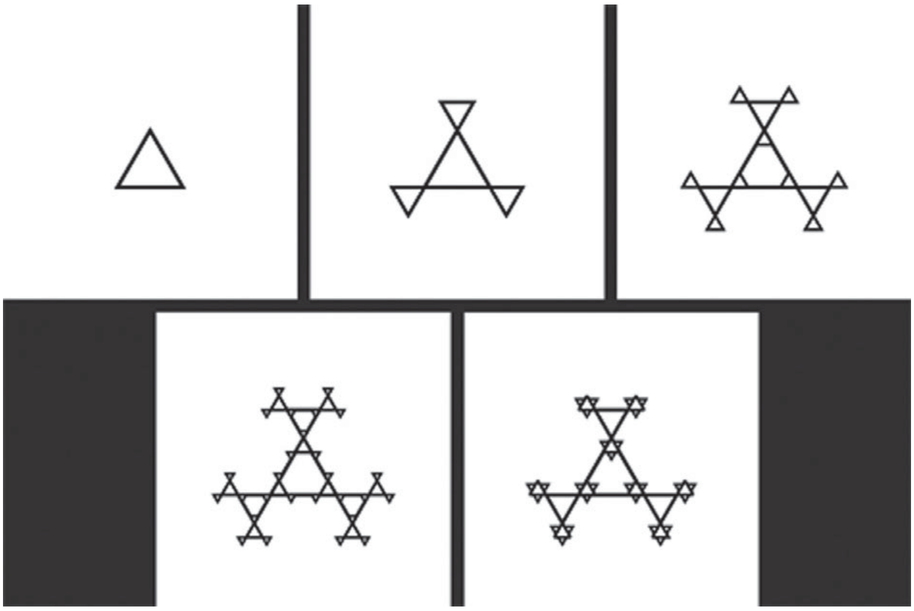

Figure 4. Example of a visual recursion task stimulus. The first three iterations of a fractal generation are presented in the top row. The subject is then asked to choose from the images in the bottom row, which corresponds to the correct fourth iteration.

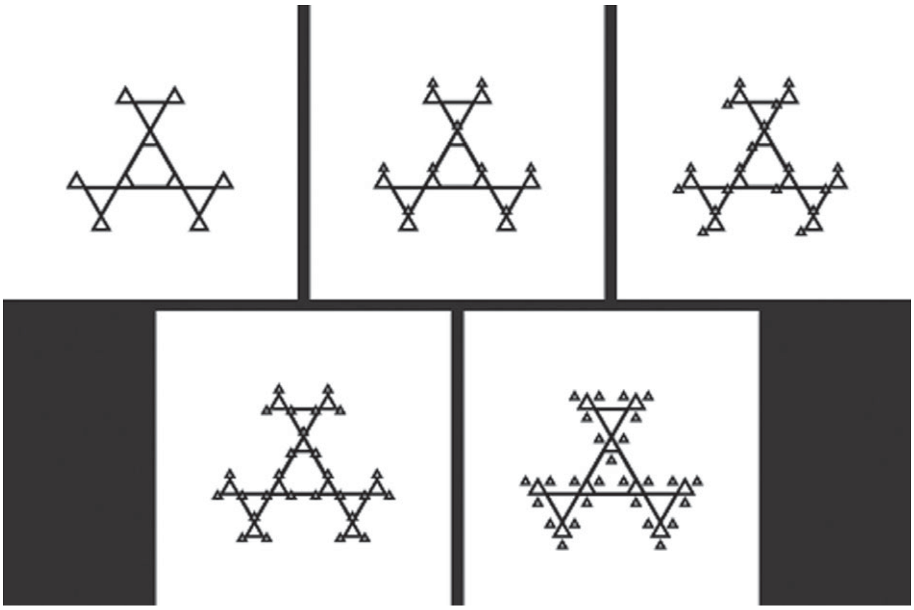

Figure 5. Example of a stimulus from the visual hierarchical task. The procedure is similar to the visual recursion task.

If these conclusions hold, the next empirical question is whether verbal processing resources are a necessary condition for recursive representations in the visual domain or whether they are recruited when available, given that they enhance reasoning in non-linguistic domains [54]. New studies are underway to assess if humans can perform above chance in VRT, under conditions of verbal and motor masking. If so, this would support the hypothesis that recursion, as an abstract representational property, can be used independently of language.

7. RECURSION AND SYNTAX

The idea of universal grammar was developed to account for two facts: (i) children are able to learn which syntactic constructions are allowed, and to apply them productively, beyond what seems possible from limited input; and (ii) there is an apparent deep syntactic similarity among different languages [57–59]. Both these facts could be explained by human cognitive biases [31,35,41,51,60,61], with origins in non-linguistic domains, provided that they would be flexible enough to be useful in a fast-changing cultural environment [50]. The ability (or predisposition) to represent hierarchical structures as containing self-similarity could be one of these biases.

A few have challenged the importance of recursion in language, claiming that it is not used in all languages [62,63] and that it is not very common in the languages where it is used [29]. These facts have led some to the conclusion that the usage of recursion is a cultural option and not a requirement for the evolution of language. Although languages are cultural conventions (despite the fact that they might recruit innate cognitive abilities [64]), I think that the importance of recursion in the processes of acquisition and development of conventionalized communication systems should not be disregarded. There are several reasons to think so: (i) children seem to be able to generalize recursive syntactic structures even though they are rare in the input. This seems to suggest that they are able to represent syntactic recursion *a priori* [28,31,52]; (ii) although recursion can be meaningfully used in prosody [32] and in discourse [30,62], its application in syntax has the property of reducing the semantic ambiguity [31]. This expressive power might have been one of the reasons why recursion became available for syntax. However, tools that enhance expressiveness and reduce ambiguity may be less necessary in conditions where the linguistic content can be predicted by the context or by a shared cultural background. This could explain why communication within communities [63] seems to be full of idioms and ungrammatical expressions [11].

The point is that the current rarity of recursive syntax in speech may not reflect its importance in the evolution and acquisition of language. The fact that languages can lose overt markers of clausal integration during glossogenetic history [30] seems to suggest that combinatoriality in syntax does not always evolve towards greater signal complexity.

8. CONCLUSION

A definition of recursion, such as 'the ability to represent a succession of hierarchical dependencies (related by parenthood) with the same rules'—by focusing on the representational and behavioural level—can be turned into empirical questions, which, provided with the appropriate methods, can be tested experimentally.

Under this definition, specific behavioural signatures of recursion can be outlined, namely the ability to take generative steps beyond the given (regarding hierarchical embedding). These traits not only distinguish recursion from hierarchical embedding and iteration, but also disqualify some abilities from being recursion-specific. For example, infinity can be also achieved by an unlimited concatenation of finite elements using iteration [38]; long-distance dependencies relate to hierarchical parsing and the usage of memory to process serial information [17,18,36,65]; and cognitive grouping can create supra-hierarchical levels with existing constituents, but cannot recruit or create new constituents.

The availability of recursion as a cognitive ability (versus hierarchical embedding) might have allowed the development of behavioural traits potentially advantageous in a wide range of domains: problem-solving, action sequencing, spatial navigation and social navigation [1,9,27,28,44,45] (J. Flack, R. Jackendoff, D. C. Krakauer & S. A. DeDeo 2011, personal communication). Furthermore, because recursion can induce cognitive biases to interpret hierarchies as self-similar—if shared by a population—these biases might have increased the likelihood of the emergence of conventionalized communication systems [50,51].

If recursion has been available in other domains before the faculty of language, it is also possible that further modifications could have occurred in the human lineage, for example, adaptations to serialized information or to symbolic representation [7,10]. In the social and spatial domains, recursion may have been used prior to the ability to represent serial and symbolic information. This does not exclude that these processes were not further enhanced by these language-specific adaptations.

Another important issue concerns the nature of recursion. Although for operational reasons we define recursion as a monolithic construct, it can be an epiphenomenon resulting from the interaction of several cognitive abilities [30,31,41]. For example, if we consider the evolution from hierarchical embedding to recursion, then there is one trait that seems to be crucial. That is the ability to compare (and match) relations between different hierarchical levels related by parenthood (level 1 is for level 2, as 2 is for level 3) and to further generalize the obtained rules to other ('mother' or 'child') levels or constituents [34]. Evidently, this entails that subjects are sensitive to the particular features targeted for the cross-level comparison, before similarity principles are extracted.

Differences between recursion and hierarchical processing can be addressed with the new methods presented in this manuscript: VRT and embedded iteration task. Given that these methods are based on the representation of visuo-spatial information, they can provide insights regarding the question of whether recursion can be used independently of language. These insights will come from research with children, verbal masking, patients with aphasia and from animal studies.

Whatever its precise role in language, recursion is an important property of human cognition and one that is used for the transmission of information. The exciting questions that it raises concerning our evolutionary history should be researched within a clear framework and one that allows the development of empirical approaches in different domains.

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